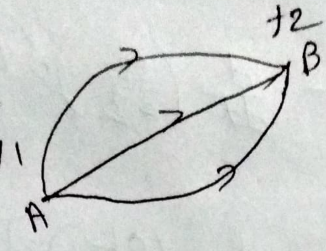
LAGRANGE EQUATION FROM HAMILTON PRINCIPLE

consider a system occupies the position A and B at time t_1 and t_2 respectively



The system, in going from A to B choose that path for which the action function has minimum value

$$\text{Action function} = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, t)\, dt.$$

Thus, for actual Path between two points, the variation in the action function should be zero.

$$= \delta\left[\int_{t_1}^{t_2} L(q_i, \dot{q}_i, t)\, dt\right] = 0$$

This is known as Hamilton principle of least action

<u>Derivation</u>

$$\int_{t_1}^{t_2} \delta L\, dt = 0$$

$$\int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta \dot{q} + \frac{\partial L}{\partial t}\delta t\right) dt = 0$$

$$\int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta \dot{q}\right) dt = 0$$

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial q}\delta q\, dt + \int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}}\delta \dot{q}\, dt = 0$$

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial q}\,\delta q\,dt + \frac{\partial L}{\partial \dot{q}}\int_{t_1}^{t_2} \delta \dot{q}\,dt - \int_{t_1}^{t_2} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q\,dt = 0$$

$\because$ There is no variation in at point A and B

$$\frac{\partial L}{\partial q}\int_{t_1}^{t_2} \delta \dot{q}\,dt = 0$$

$$\int_{t_1}^{t_2} \left[\frac{\partial L}{\partial q}\delta q - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\delta q\right]dt = 0$$

$$\int_{t_1}^{t_2} \left[\frac{\partial L}{\partial q}\delta q - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right]\delta q\,dt = 0$$

Since δq is arbitrary therefore

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0}$$

This is the required Lagrange equation of motion.